2901/201
APPLIED MATHEMATICS
June/July 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN PETROLEUM GEOSCIENCE

MODULE II

APPLIED MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination
Mathematical tables / a non programmable scientific calculator (fx-82);
An abridged table of Laplace Transforms;
The Standard Normal Distribution and the X^2 Distribution tables are attached.
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Marks for each part of a question are indicated.
Candidates should answer the questions in English.
Candidates should indicate the questions they have answered in the answer booklet.

This paper consists of 7 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Determine the Laplace transform of $f(t) = e^{2t} \sin 3t$ from first principles. (9 marks)

(b) Use Laplace transforms to solve the differential equation

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 25y = te^{-5t} \quad \text{at } t = 0,\ y = 1,\ \frac{dy}{dt} = 3$$

(11 marks)

2. (a) Given the Matrix $M = \begin{bmatrix} 3 & -6 & 2 \\ 6 & 2 & -3 \\ 2 & 3 & 6 \end{bmatrix}$

(i) Show that $MM^T = \lambda I$ where I is an identity matrix.

(ii) Hence determine M^{-1}.

(7 marks)

(b) A manufacturer produces an alloy made from steel, aluminium and copper. The cost of 2 tonnes of steel, 4 tonnes of aluminium and 3 tonnes of copper is Ksh. 3,250,000, the cost of 1 tonne of steel, 4 tonnes of aluminium and 5 tonnes of copper Ksh. 4,400,000 while the cost of 6 tonnes of steel, 7 tonnes of aluminium and 3 tonnes of copper is Ksh. 4,600,000. Use the inverse matrix method to determine the cost of each metal.
(13 marks)

3. (a) Given that $Z = e^{(6x+5)} Sin\,(6y + 8) + 4x + 3y + 9$

Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

(4 marks)

(b) The pressure P and volume V of a gas are related by the equation $PV^{1.4} = C$

Determine the approximate percentage change in C when the pressure is increased by 2.3% while the volume is decreased by 0.84% using partial differentiation.
(7 marks)

(c) Locate the stationary values of the function

$$f(x,y) = \frac{x^3}{x} + \frac{5x^2}{2} - 6x + 4y^2 + 7$$

(9 marks)

4. (a) Water at a temperature of 100 °C cools to 88 °C in 10 minutes at room of temperature of 25 °C. Use the Newton's Law of cooling to determine the temperature of the water after 20 minutes. (8 marks)

(b) Solve the simultaneous differential equations.

$$\frac{dx}{dt} = 3x + 8y$$

$$\frac{dy}{dt} = -x - 3y$$

given that when $t = 0$, $x = 6$ and $y = -2$

(12 marks)

5. (a) (i) Determine the MacLaurin's series expansion of

$f(x) = xe^{2x}$ up to the term in x^5

(ii) Hence evaluate the integral $\int_0^1 \frac{xe^{2x}}{x} dx$

(11 marks)

(b) (i) Use Taylor's theorem to expand $f(x) = Cot(x + h)$ as far as the term in h^3.

(ii) Hence determine $Cot\, 46°$.

(9 marks)

6. (a) A particle moves the curve

$\underline{\gamma} = (t^3 - 4t)\underline{i} + (t^2 + 4t)\underline{j} + (8t^2 - 3t^3)\underline{k}$ where t is the time.

Determine the magnitude of the tangential component of its acceleration at $t = 2$.

(10 marks)

(b) Given the surfaces $P = x^2 + y^2 + z^2 - 9$ and $Q = Z - x^2 - y^2 + 3$ at point $(2, -1, 2)$.

Determine:

(i) unit vectors normal to P and Q;

(ii) angle between the surfaces.

(10 marks)

7. (a) Table I shows the number of litres of kerosene sold by two petrol stations during the covid-19 pandemic.

Table I

Station A	1	3	5	7	8	10	12	15	17	20
Station B	8	12	15	17	18	20	22	23	24	25

Use the assumed mean of station A as 7 and station B as 15 to calculate the coefficient of correlation. (8 marks)

(b) Table II shows the marks scored by 50 students in a maths exam.

Table II

Marks	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
No. of students	5	*a*	7	6	*b*	3	5	3	3

Given that the mean mark is 59.9 determine the:

(i) values of ***a*** and ***b***;

(ii) median mark.

(12 marks)

8. (a) Seven percent of resistors produced by a machine are defective. In a random sample of 8 resistors, determine the probability that:

(i) none are defective.;

(ii) at most **three** are defective.;

(iii) at least **four** are defective.

(8 marks)

(b) A continuos random variable T has a probability density function defined by:

$$f(t)=\begin{cases} k(t^2+ct), & 0\le t\le 3 \\ 0, & elsewhere \end{cases}$$

where ***k*** and ***c*** are positive constants.

Given that the mean is $\frac{21}{10}$ determine:

(i) the values of ***k*** and ***c*** ;

(ii) the standard deviation.

(12 marks)