2310/301
2311/301
2312/301
2313/301
MATHEMATICS
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN PHOTOGRAMMETRY AND REMOTE SENSING
DIPLOMA IN CARTOGRAPHY
DIPLOMA IN LAND SURVEYING
DIPLOMA IN MAP REPRODUCTION

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
 answer booklet;
 mathematical tables/ scientific calculator.
*Answer **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are indicated.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all
the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Simplify the expression:

$$\frac{(\cos 5\theta + j\sin 5\theta)^{-4}(\cos 7\theta - j\sin 7\theta)^8}{(\cos 2\theta + j\sin 2\theta)^6(\cos 4\theta +)\sin 4\theta)^6}$$ (4 marks)

   (b) Solve the equation:

$$z^4 - 5j - 12 = 0$$ (9 marks)

   (c) Given that $z = x + yi$ determine the locus of $\arg\left[\dfrac{x + 2j}{z - 2j}\right] = \dfrac{\pi}{4}$. (7 marks)

2. (a) (i) Given the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ show that $A^3 - 6A^2 + 6A - 11I = 0$ where I is an identify matrix.

      (ii) hence determine A^{-1}. (10 marks)

   (b) (i) Given the matrix $M = \begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$, show that $MM^t = KI$ where K is a constant :

      (ii) Hence, determine M^{-1}.

      (iii) Use the result in (ii) to solve the equations

$$-2x + y + z = 18$$
$$2x + 2y + z = 10$$
$$x - 2y + 2z = 5$$ (10 marks)

3. (a) If $\sin A = \dfrac{5}{13}$ and $\cos B = \dfrac{4}{5}$ where A and B are acute angles, determine the values of:

$$\cos(A - B)$$ (5 marks)

   (b) Solve the equation: $4\sin^2\theta - 12\sin 2\theta + 35\cos^2\theta$ for values θ between $0°$ and $90°$. (6 marks)

   (c) Given that A, B and C are angles of a triangle, prove that:

$$\cos 4A - \cos B - \cos 4C = 4\sin 2B \sin 2C \cos 2A - 1.$$ (9 marks)

4. (a) Given that $V = \ln\left(x^2 + y^2\right)^{\frac{1}{7}}$. Show that $\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} = 0$. (5 marks)

(b) The radius of a cone increases by 0.2 cm while the height decreases by 0.3 cm. Use partial differentiation to determine the change in the surface area of the cone when the radius is 5 cm and the height is 12 cm. (6 marks)

(c) Locate the stationary points of the function $f(x, y) = 2x^2 + 7xy - 76x + 4y^2 - 99y + 40$ and determine their nature. (9 marks)

5. (a) Use the Newton-Raphson method to solve the equation $x^3 - 7x^2 + 5x - 4 = 0$ near $x_0 = 5$ correct to eight decimal places. (11 marks)

(b) Use Simpon's rule with four strips to evaluate the integral $\displaystyle\int_0^\pi \frac{2d\theta}{\sqrt[4]{5 - \dfrac{3}{8}\cos^3\theta}}$ correct to six decimal places. (9 marks)

6. (a) Solve the differential equation:

$$\left(y\sec^2 x + \sec x \tan x\right)dx + \left(\tan x + 2y\right)dy = 0$$ (6 marks)

(b) Use the method of undetermined coefficients to solve the differential equation:

$$\frac{d^2 y}{dt^2} - 4\frac{dy}{dt} + 13y = 4\cos t \text{ given that when } t = 0,\ y = \frac{181}{145} \text{ and } \frac{dy}{dt} = 2.$$ (14 marks)

7. (a) Table 1 shows the performance of 8 land survey students in a test and main exam.

Table 1

Test	17	16	19	20	24	25	18	13
Main exam	26	23	22	20	26	19	24	16

Determine the coefficient of correlation between the test and main exam marks. (9 marks)

(b) A continuous random variable X with a probability density function (p.d.f) is given by:

$$f(x) = \begin{cases} k_1 x, & 1 \le x \le 3 \\ k_2(4 - x), & 3 \le x \le 4 \\ 0, & elsewhere \end{cases}$$

where k_1 and k_2 are constants.

Determine the:

(i) values of k_1 and k_2;
(ii) mean of x. (11 marks)

8.   (a)   Determine the value of n if $^np_4 = 30\,{}^nc_5$.   (4 marks)

      (b)   (i)   Use the binomial theorem to expand $\sqrt{\dfrac{3+4x}{3-4x}}$ up to the third term.

             (ii)   By setting $x = \dfrac{1}{64}$ in the result in (i), determine $\sqrt{47}$ correct to 3 decimal
                    places.   (11 marks)

      (c)   Show that the polar equation of the curve $4y^2 - 4x + 1 = 0$ is $r = \dfrac{1}{2(1 - \cos\theta)}$.

                                                                               (5 marks)
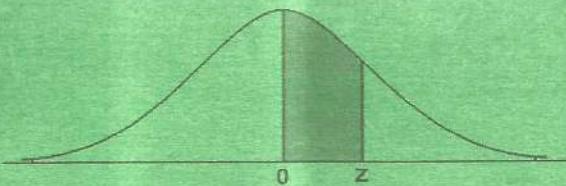
Normal Probability

Z	Area under the standard normal curve from 0 to Z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

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