2310/301
2311/301
2312/301
2313/301
MATHEMATICS
June/July 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN PHOTOGRAMMETRY AND REMOTE SENSING
DIPLOMA IN CARTOGRAPHY
DIPLOMA IN LAND SURVEYING
DIPLOMA IN MAP REPRODUCTION

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/ Scientific calculator.
*Answer **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) The third term of a geometric progression is eight time the sixth term and the sum of the first four terms is 45/8. Determine the:

(i) common ratio;

(ii) first term;

(iii) sum to infinity of the progression. (11 marks)

(b) (i) Use Taylor's theorem to expand $\sin\left(\frac{\pi}{6}+h\right)$ as far as the term in h^3.

(ii) Determine the value of $\sin 31\frac{1}{2}^\circ$ using the result in (i) above, correct to four decimal places.

(9 marks)

2. (a) Given the matrices

$$A=\begin{bmatrix}1 & -1 & 2\\ -1 & 1 & 3\\ 1 & 0 & -1\end{bmatrix}, \quad B=\begin{bmatrix}1 & 1 & -1\\ -1 & 1 & 2\\ 0 & 1 & -1\end{bmatrix}$$

(i) determine a matrix $C = 2A + B$

(ii) show that $(BA)^T = A^T B^T$.

(9 marks)

(b) Use the inverse matrix method to solve the simultaneous equations

$$x+y-z=0$$
$$y-z=-5$$
$$-x+y+2z=-1$$

$x+y-z=0$

$z_2+z_3=$

(11 marks)

3. (a) Given the complex numbers $z_1 = 2-3j$, $z_2 = 1+j$ and $z_3 = -1+j$, determine the complex $z = z_1 + \frac{z_2 z_3}{z_2+z_3}$ giving your answer in polar form. (8 marks)

(b) Use De Moivre's theorem to show that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$. (5 marks)

(c) Solve the equation $z^2 - 8 + 8j\sqrt{3} = 0$, giving the answer in polar form. (7 marks)

Inverse

$$\begin{bmatrix}1 & 1 & -1\\ 1 & 1 & -1\\ -1 & 1 & 2\end{bmatrix}$$

$\det = 1\begin{vmatrix}1 & -1\\ 1 & 2\end{vmatrix} - 1\begin{vmatrix}1 & -1\\ -1 & 2\end{vmatrix} + (-1)\begin{vmatrix}1 & 1\\ -1 & 1\end{vmatrix}$

$1(2+1) - 1(2-1) + (-1)(1+1)$

$3-1-2$

$z = a+ib$

$(2+i3)+(4+i5)$

$(2+4)+i(3+5)$

$6+i8$

$z = z_1 + \frac{z_2 z_3}{z_2+z_3}$

$z = 2-3j + \frac{(1+j)(-1+j)}{1+j+-1+j}$

Denominator

$(1+j)+(-1+j)$

$(1-1)+j(1+1)$

SOHCAHTOA

$z = 2-3j + \frac{(1+j)(-1+j)}{2j}$

4. (a) (i) Given that x_n is an approximation to the root of the equation $x^3 + x - 6 = 0$, use the Newton-Raphson method to show that a better approximation is given by $x_{n+1} = \dfrac{2x_n^3 + 6}{3x_n^2 + 1}$.

(ii) Taking $x_o = 1.5$, determine the root in (i) above, correct to four decimal places. (10 marks)

(b) Use Simpson's rule with six intervals to determine the value of the integral

$\int_1^{1.6} \ln x \, dx$, correct to three decimal places. (10 marks)

5. (a) Show that the solution of the differential equation $y\dfrac{dy}{dx} + \dfrac{1+y^2}{1+x^2} = 0$, given that $y = 0$ when $x = 0$ may be expressed in the form $y^2 = e^{-2\tan^{-1}x} - 1$. (8 marks)

(b) Use the method of undetermined coefficients to solve the differential equation

$\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 4y = e^{-2x}$, given that when $x = 0$, $y = 0$ and $\dfrac{dy}{dx} = 0$. (12 marks)

6. (a) Find $\dfrac{dy}{dx}$, given

(i) $x^2 - 2y^2 + 4xy - 6x = 3$

(ii) $y = \sin\left(\dfrac{x+1}{x+2}\right)$.

(9 marks)

(b) The radius of a cylinder is changing at the rate of 3 cm/s and its height is changing at the rate of 6 cm/s. Use the partial differentiation to determine the rate at which the volume of the cylinder is changing at the instant when its radius is 12 cm and its height is 18 cm. (4 marks)

(c) Locate the stationary points of the function $f(x, y) = x^2 - 3y^2 + 6\,xy - 24x$, and determine their nature. (7 marks)

7. (a) (i) Evaluate the integral

$$\int_0^1 \frac{x\,dx}{(x+1)^2(x+2)}$$

(ii) Use the substitution $u = x^2$ to show that $\int_0^1 \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}}\,dx = \pi^2/16$.

(14 marks)

(b) Use integration to determine the area of the region bounded by the curves $y = x^2$ and $x = y^2$.

(6 marks)

8. (a) A manufacturing company employes 100 artisans, 70 technicians, 25 technologists and 5 engineers. Represent the data using a pie chart.

(5 marks)

(b) A random variable x is normally distributed with a mean of 38 and a standard deviation of 3. Determine the probability that x lies between 36 and 39.

(6 marks)

18%
100/200 × 360
18
70/200 × 360
126%
25/200 × 360 45%
5/200 × 360 9%

(c) A sample of 9 nails from a production line has a mean length of 15.0 cm and a standard deviation of 3.0 cm. Given that the lengths follow a normal distribution, determine the:

(i) point estimate of the:

(I) mean;

(II) standard deviation.

(ii) 95% confidence interval of the mean.

(9 marks)

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