

2305/301, 2308/301
2306/301, 2309/301
2307/301

MATHEMATICS
Oct./Nov. 2016
Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

**DIPLOMA IN BUILDING
DIPLOMA IN QUANTITY SURVEYING
DIPLOMA IN CIVIL ENGINEERING
DIPLOMA IN HIGHWAY ENGINEERING
DIPLOMA IN ARCHITECTURE**

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet

Mathematical tables/calculator

Drawing instruments

*Answer **FIVE** questions of the **EIGHT** questions in the answer booklet provided.*

All questions carry equal marks.

Maximum marks for each part of a question are shown.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

- 1/ (a) Given that $Z_1 = 3 + 2j$; $Z_2 = -3 + 5j$ and $\frac{1}{Z^3} = \frac{1}{Z_1} + \frac{1}{Z_2}$; determine Z_3 in the form $a + bj$. (4 marks)
- (b) Use De Moivre's theorem to find the following in terms of $\cos \theta$ and $\sin \theta$:
- (i) $\cos 3\theta$
 - (ii) $\sin 3\theta$ and hence
 - (iii) Show that $\tan 3\theta = \frac{3t - t^3}{1 - 3t^2}$ where $t = \tan \theta$. (6 marks)
- (c) Given that $Z^3 = -8 - 8\sqrt{3}j$, find Z in the form $a + bj$. (10 marks)

- 2/ (a) Given that $A = pi - 6j - 3k$ and $B = 4i + 3j - k$, where p is a constant. Determine the value of p such that vectors A and B are perpendicular to each other. (4 marks)
- (b) Determine the area of the triangle whose two sides are the vectors:
 $A = 3i - 2j + 4k$
 $B = i + 5j - 2k$
 (Handwritten notes: $\begin{matrix} p & -6 & -3 \\ 4 & 3 & -1 \end{matrix}$, $\sqrt{4^2 + 13^2 + 1^2} = \sqrt{p^2 + 16 + 3^2}$, $5.099 = \sqrt{p^2 + 16}$) (6 marks)
- (c) Given that $6 \sin \theta - 8 \cos \theta = R \sin(\theta - \alpha)$ where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$.
- (i) Determine the values of R and α and hence
 - (ii) Solve the equation $6 \sin \theta - 8 \cos \theta = 5$ where $0^\circ \leq \theta \leq 360^\circ$. (10 marks)

- 3/ (a) Given that $V = x^2 \cos\left(\frac{y}{x}\right)$, show that (Handwritten note: UATE)
 $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} = 3V$ (5 marks)
- (b) A right circular cylinder has radius, $r = 10$ cm and height, $h = 100$ cm. If the radius is increasing at the rate of 0.5 cm/s, determine the rate at which the volume is changing. (Handwritten note: $V = \pi r^2 h$) (5 marks)
- (c) Given that $t = e^{2h} \ln(4h + 5p)$. Find the change in the value of t if h is increased from 0.4 cm to 0.44 cm and p is decreased from 0.3 cm to 0.28 cm. (10 marks)

4. (a) (i) Given that $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$, evaluate $\int_0^{\frac{\pi}{2}} \cos^4 \theta \, d\theta$, giving your answer correct to two decimal places.
- (ii) Evaluate $\int_2^3 \frac{5x^2 + 15x - 2}{(x-1)(x+2)^2} \, dx$ (13 marks)
- (b) (i) Determine the area of the region enclosed by $y = 6 \sin 3x$, and x-axis between $x = 0$ and $x = \frac{\pi}{3}$.
- (ii) Find the volume generated when the area in b(i) is rotated about the x-axis through 360° . (7 marks)

5. (a) Solve the differential equation $2xy \frac{dy}{dx} = x^2 + y^2$, given that when $x = 3$, $y = 2\sqrt{2}$. (8 marks)
- (b) Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4e^{2x}$, given that when $x = 0$, $y = 2$ and $\frac{dy}{dx} = 0$. (12 marks)

6. (a) Given that $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & -1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 1 \\ 1 & -2 & -3 \\ 3 & -1 & -2 \end{bmatrix}$
- Show that $(AB)^T = B^T A^T$ (7 marks)
- (b) Use inverse matrix method to solve the simultaneous equations
- $$\begin{aligned} 2x - 3y + 2z &= 9, \\ 3x + 2y - z &= 4, \\ x - 4y + 2z &= 6. \end{aligned}$$
- (13 marks)

7. (a) A random sample of size 100 from an infinite population has a mean of 80 and a standard deviation of 15. Determine the probability that the sample mean lies between 79 and 82. (6 marks)

Handwritten notes:
 $p = 0$
 $\text{sum} = 0$
 $\frac{AV}{TB}$
 $(\text{sum}) / (\text{sum})$
 $V = (1+V)(0-V)$
 $1 - V = V$
 $0 = V + 2 - V^2$
 $(V+1)(2-V)$
 $-V^2 + 2V - 2$
 $-V^2 + 2V - 2$
 $-V^2 + 2V - 2$

- (b) A continuous Random variable x has probability density function defined by

$$f(x) = \begin{cases} Kx^2; & 0 < x < 1 \\ K; & 1 < x < 2 \\ 0; & \text{elsewhere} \end{cases}$$

Determine:

- (i) the value of the constant K .
- (ii) the mean and standard deviation of x .
- (iii) $P(x < \frac{1}{2})$

(14 marks)

8. (a) Two major brands of light bulbs A and B are used in an apartment. A sample of 50 bulbs of brand A revealed a mean life-time of 6.94 months and a standard deviation of 0.82 months. A sample of 60 bulbs from brand B revealed a mean life time of 7.34 months and a standard deviation of 1.53 months. Test at 5% level of significance the claim that there is no difference in quality between the two brands of bulbs.

(7 marks)

- (b) An experiment was carried out on small cantilevered steel beam, various masses were placed on the end of the beam and corresponding deflections measured as shown in table 1.

Table 1

Mass x (grammes)	Deflection y (mm)
0	0
50.15	0.6
99.90	1.8
150.05	3.0
200.05	3.6
250.20	4.8
299.95	6.0
350.05	6.2
401.00	7.5

- (i) Find the least squares line of Regression of y and x .
- (ii) Predict the deflection when the mass is 220g.

(13 marks)

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