2405/301 MATHEMATICS Oct./Nov. 2016 Time: 3 hours





THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN APPLIED STATISTICS

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Scientific calculator
This paper consists of EIGHT questions.
Answer any FIVE questions in the answer booklet provided.
ALL questions carry equal marks.
Maximum marks for each part of a question are indicated.
Candidates should answer all questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

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1. (a) Given the matrices

$$x = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ -4 & -1 & 2 \end{pmatrix}, \quad y = \begin{pmatrix} 2 & 4 & -3 \\ 5 & 8 & 1 \\ 3 & -2 & 7 \end{pmatrix} \quad and \quad z = \begin{pmatrix} 1 & 2 & -4 \\ -6 & 3 & 5 \\ 7 & 2 & -9 \end{pmatrix}$$

Find

- (i) y2
- (ii) zyz

- 30 = 80 + 45 + 2c 30 = 40 + 35 + c10 = 20 + 5 + 367 marks
- (b) In the production of various cereals, fertilizers a wand c are mixed in various combinations. To produce 30 tonnes of barley 8 units of a, 4 units of b and 2 units of c are used. For wheat, 4 units of a, 3 units of b and 1 unit of c are applied to produce 30 tonnes while for maize, 2 units of a, 1 unit of b and 3 units of c produce 10 tonnes. Using the inverse matrix method, find the quantity of each fertilizer that is required to produce the combined harvest. (13 marks)
- 2. (a) Find the differential equation with the given form of general solution

$$y = Ae^{3x} + Bxe^{3x}$$

(7 marks)

(b) Solve the following ordinary differential equations

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-xt}$$

given that at t = 0, $x = \frac{dx}{dt} = -2$

(13 marks)

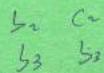
(a) Given that $U = l_n(x^2 + y^2)$ Show that

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$$

(7 marks)

- (b) Given that $z = e^u \sin 2v$, if u is increasing at 3 units per second and v is decreasing at 5 units per second, determine the rate at which z is changing when u = 0.1 unit and v = 0.5 unit. (7 marks)
- (c) The radius of a right cone is increased from 6 cm to 6.2 cm and its height is reduced from 10 cm to 9.9 cm. Use partial derivatives to determine the change in volume of the cone. (6 marks)

2405/301 Oct./Nov. 2016 DV (ON) + DU (Oh.).



Given that z' = -16, find all possible values of z in the form a + bj(a)

(8 marks)

- Use Demoivre's theorem to show that $Sin^{\epsilon}\theta = \frac{1}{32}(10-15\cos 2\theta + 6\cos 4\theta \cos 6\theta)$ (b) (6 marks)
- (c) Given that

$$z_1 = 10 - 5j$$

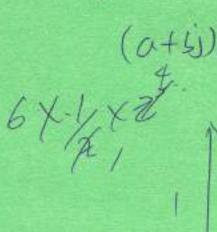
$$z_1 = 4 + 6j$$

$$z_3 = 2 - 7j$$

Find

- (i) 2725
 - $w = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ (ii)

in the form a + bj

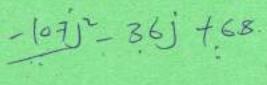


(6 marks)

- Use Taylor's theorem to expand $Sin(\sqrt[n]{+}h)$ in ascending powers of h up to (a) (i) and including h'.
 - (ii) (11 marks)
- (b) (i) Determine the first four terms of the Maclaurin's series expansion of $f(x) = l_n(1+4x)$.
 - (ii) Hence evaluate

$$\int\limits_0^1 \frac{1}{x} \, l_\pi \left(1 + 4x\right) dx$$

giving the answer correct to four decimal places.



(9 marks)

6. Evaluate (a)

$$\int_{0}^{1} \int_{0}^{\sqrt{1+x^2}} (x^2y^3 + 3y) \, dy \, dx$$

(1+4x) (4-(1+4x) = 4.9 marks)

Change the order of integration and evaluate (b)

$$\int_{0}^{2} \int_{0}^{x} (3x + 2x^{2}y^{2}) dy dx$$

(7 marks)

2405/301 Oct/Nov. 2016

Turn over

- Use double integrals to determine the area bounded by the graphs y = 0, x = 0 and (0) y = 2 - x(4 marks)
- 7. (a) Prove that

$$\frac{\tan x + Sec x}{Sec x (1 + \frac{\tan x}{Sec x})} = 1$$

(5 marks)

- Given a trigonometrical function $y_1 = 2 \cos 3\theta \sin \theta$, (b)
 - rewrite the function as difference of two trigonometric ratios hence draw its (1) 0.4054. graph for $-90^{\circ} \le \theta \le 90^{\circ}$.
 - (ii) solve for $y = Cos\theta$, using the graph.

(15 marks)

Show that x=2 is an approximate solution of the equation e'=4x. Hence apply the (a) Newton Raphson method to calculate the root correct to four decimal places.

(9 marks)

(b) The function f(x) is tabulated in table 1. ex=4x ex=0

Table 1

X	-8	-60	-4	-2	0	2
f(x)	-2167	927	-279	+31	9	33

Use Newton-Gregory formulae to calculate

- (i) 1(-7.5)
- f(1) (ii)

egory formulae to calculate fx = fp + f fx + f(f-1)fx $fx = fp + f fx + f(f+1) + fx^2 + ff f(f+1) + fx^2 + f$

90 180