1521/203
1601/203
1602/203
MATHEMATICS II
Oct./Nov. 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN ELECTRICAL AND ELECTRONIC TECHNOLOGY
(POWER OPTION)
(TELECOMMUNICATION OPTION)
MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/calculator.
*Answer any **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
All necessary working must be clearly shown.
Maximum marks for each part of a question are as indicated.
Candidates should answer all questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Prove the identities:

(i) $\frac{1+\sin\theta}{\sin\theta\cos\theta}=\tan\theta+\cot\theta+\sec\theta$

(ii) $\sec^4\theta-\sec^2\theta=\tan^2\theta+\tan^4\theta$

(6 marks)

(b) Solve the equations:

(i) $3\tan^2\theta+5=7\sec\theta$

(ii) $4\cos\theta-3\sin\theta=2.5$

for values of θ between 0° and 360° inclusive. (14 marks)

2. (a) Determine the values of the constants P and Q in the equation

$$P\cosh 3x-Q\sinh 3x=e^{3x}+5e^{-3x}$$

(5 marks)

(b) Prove the identities by using exponential definition:

(i) $1-\tanh^2x=\operatorname{sec}h^2x$

(ii) $1+\operatorname{cosec}h^2x=\coth^2x$

(6 marks)

(c) Solve the equations:

(i) $\cosh 2x-3\sinh x=0$

(ii) $3\tanh^2x=5\operatorname{sec}h\,x+1$

correct to four decimal places. (9 marks)

3. (a) Find $\frac{dy}{dx}$ for the functions:

(i) $y=x\sin^2(3x)$

(ii) $y=\frac{e^{2x}}{(x+1)}$

(6 marks)

(b) Given that $z=\cos(xy)$, show that

$$\frac{\partial^2z}{dy^2}=-(x^2+y^2)z$$

(5 marks)

(c) A solid cylinder of radius r cm and height h cm has a volume, V of 500 cm^3.

(i) Express the total surface area, A cm^2 in terms of r.

(ii) Determine the value of r for which A will be minimum correct to one decimal place.

(9 marks)

4. (a) Given that

$(1+kx)^n = 1 + 48x + 1008x^2 + \ldots$ where k is a constant, determine the value of:

(i) n

(ii) k

(7 marks)

(b) (i) Determine the first three terms in the binomial expansion of $(1+x)^{\frac{1}{2}}$.

(ii) Hence by setting $x = \frac{1}{16}$ in b(i), estimate the value of $\sqrt{17}$ correct to three decimal places.

(6 marks)

(c) Determine the term independent of x in the binomial expansion of $(x^2 + \frac{1}{x})^{12}$.

(7 marks)

5. (a) Given the position vectors of the vertices of a triangle ABC as

$\underset{\sim}{A} = 20\underset{\sim}{i} - \underset{\sim}{j},\ \underset{\sim}{B} = 30\underset{\sim}{i}$ and $\underset{\sim}{C} = 10\underset{\sim}{i} + 5\underset{\sim}{j}$.

Determine the:

(i) angle between $\underset{\sim}{AB}$ and $\underset{\sim}{AC}$.

(ii) area of the triangle ABC.

(10 marks)

(b) Given that a vector field $\underset{\sim}{E} = x^2y\underset{\sim}{i} + xz^2\underset{\sim}{j} + xy^2z\,\underset{\sim}{k}$ and a scalar field $\phi = x^2y^2z^2$, determine at (1, 1, 1) the values of:

(i) $|\nabla\phi|$;

(ii) $\nabla \cdot \underset{\sim}{E}$;

(iii) $\nabla \times \underset{\sim}{E}$.

(10 marks)

6. (a) Three forces F_1, F_2 and F_3 acting at a point satisfy the simultaneous equations:

$$F_1 + 2F_2 + F_3 = 8$$
$$2F_1 - F_2 + F_3 = 3$$
$$F_1 + F_2 - 2F_3 = -3$$

Solve the equations using substitution method. (10 marks)

(b) Evaluate the integrals:

(i) $\int_0^{\pi} (\cos 2x - \sin 3x)\, dx$

(ii) $\int_0^{4} (\sqrt{x} + 1)\, dx$.

(5 marks)

(c) Use integration to determine the area of the region bounded by the curve $y = 2x - x^2$, $x-$ axis and $x = 0$ and $x = 2$ (5 marks)

7. (a) Given the matrix

$$M = \begin{bmatrix} (k-3) & -2 & 0 \\ 1 & k & -2 \\ -1 & -2 & -2 \end{bmatrix}$$

where k is a constant, determine the possible values of k such that the matrix M is singular. (10 marks)

(b) Three currents I_1, I_2 and I_3 in amperes satisfy the simultaneous equations:

$$3I_1 + 2I_2 + I_3 = 7$$
$$I_1 - 2I_2 - I_3 = 1$$
$$I_1 + 3I_3 = 11$$

Use inverse matrix method to solve for I_1, I_2 and I_3. (10 marks)

8. (a) Given the matrix

$$B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}, \text{ show that } 1 - 7B + 6B^2 - B^3 = 0$$

where I is an identity matrix. (9 marks)

(b) Three emfs E_1, E_2 and E_3 in an inductive circuit satisfy the simultaneous equations:

$E_1 + 3E_2 + 2E_3 = 14$
$2E_1 + E_2 + E_3 = 7$
$3E_1 + 2E_2 - E_3 = 7$

Use Cramer's rule to solve for E_1, E_2 and E_3 in volts. (11 marks)

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