

073206T4CEN

BUILDING TECHNICIAN LEVEL 6

CON/OS/BUT/CC/1/6

APPLY ENGINEERING MATHEMATICS

March /April 2023



THE KENYA NATIONAL EXAMINATIONS COUNCIL

WRITTEN ASSESSMENT

Time: 3 Hours

INSTRUCTIONS TO CANDIDATE

- i. This assessment has two sections **A and B***
- ii. You are provided with a separate answer booklet.*
- iii. Do not write on the question paper*
- iv. Marks for each question are indicated in the brackets.*

This paper consists of SIX (6) printed pages.

The candidate should check the question paper to ascertain that all pages are printed as

indicated and that no pages are missing

SECTION A: (40 MARKS)

Attempt **THREE** questions in this section

- 1) Solve the following logarithmic equation.

$$\log_2(2z+1) = 2 + \log_2 z \quad (3 \text{ marks})$$

- 2) Solve the following trigonometric equation in the range given.

$$2 \sin \theta = 5 \cos \theta, \quad 0^\circ \leq \theta \leq 360^\circ \quad (4 \text{ marks})$$

- 3) Determine the value of x and y in the following equation. (4 marks)

$$(x + yj)(3 + 4j) = 3 - 4j$$

- 4) Determine the first three terms in the expansion $(2 - 7x)^6$ in ascending powers of x without using a calculator. (3 marks)

- 5) A water tank has a square base of length 48cm and height 35cm. It is filled with water to a height of 25cm. When a solid sphere is placed in the tank the water level raises by π cm. Assuming the sphere is fully submerged determine the radius of the sphere. (4 marks)

- 6) Determine the mean of the following data

Class interval	0-10	10-20	20-20	30-40	40-50
frequency	3	5	8	3	1

(4 marks)

- 7) Evaluate $\int \sin^2 x dx$ (2 marks)

- 8) The following vectors are given

$$\vec{a} = 2i + 3j - k$$

$$\vec{b} = i + 2j + k$$

$$\vec{c} = j + 3k$$

- a) Show the three vectors are coplanar.

- b) Express \vec{a} in terms of \vec{b} and \vec{c} . (4 marks)

- 9) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ Determine whether AB is singular or non – singular

(3 marks)

10) Convert the polar equation $r = \cos \theta + \sin \theta$, $0 \leq \theta \leq 2\pi$. Hence show it represents a circle by determining its coordinates and radius. (4 marks)

11) Determine the solution of the first order differential equation $\frac{dy}{dx} + \frac{4y}{x} = 6x - 5$. Subject to the boundary condition at $y = 1$, at $x = 1$.

(5 marks)

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SECTION B: (60 MARKS)

Attempt any **THREE** questions. All questions carry **equal** marks

- 12) a) Solve the second order differential equation using the method of undetermined coefficient.

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 10\sin x \quad \text{Given the boundary condition } y = 6, \quad \frac{dy}{dx} = 5, \quad \text{at } x = 0$$

(12 marks)

- b) Find the Maclaurin's expansion of $\ln\left(\sqrt{\frac{1+2x}{1-2x}}\right)$, up and including the term x^3 .

(8 marks)

- 13) a) Given the equation $x^2 = \frac{2}{\sqrt{x}} + \frac{3}{x^2}$

- i. Show that the root lies between 1 and 2.
- ii. Use Newton Raphson method to show that the root can be found by the iterative

$$\text{formulae } x_{n+1} = \frac{x_n^5 + 3x_n^{\frac{3}{2}} + 9x_n}{2x_n^4 + x_n^{\frac{3}{2}} + 6}$$

- iii. Hence determine the root correct to 8 decimal places. (10 marks)

- b) Solve the following hyperbolic equation $2\cosh 2x + 10\sinh 2x = 5$. (10 marks)

- 14) a) Use Demoivre's theorem to prove the validity of the following trigonometrical identity

$$\sin 5\theta = \sin \theta (16\cos^4 \theta - 12\cos^2 \theta + 1) \quad \text{Hence or otherwise solve the equation}$$

$$\sin 5\theta = 10\cos \theta \sin 2\theta - 11\sin \theta, \quad 0 \leq \theta \leq \pi \quad (10\text{marks})$$

- b) A right circular cylinder has radius 5cm and height 10cm. Use partial differentiation to determine the approximate increase in the volume of the cylinder if the radius increase by 0.4 cm and its height decreases by 0.2cm. (5 marks)

- c) The length of reinforcement bars is normally distributed with a mean of 115 cm and standard deviation of 2,6. Determine the that the length of a bar picked at random will be between 10cm and 14cm. (5 marks)

- 15) a) The 3×3 matrix A is given below

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

- i. Determine the inverse of A.
- ii. Hence solve the system of the equation

$$x + 2y + z = 1$$

$$2x + 3y + z = 4 \quad (10\text{marks})$$

$$3x + 4y + 2z = 4$$

- b) Determine the area bounded by the curves $y = 3x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$
(5marks)

- c) Given that for all values of x $.5x^2 + Ax + 7 = B(x - 2)^2 + c$. Determine the values of each of the Values of the constant.
(5marks)

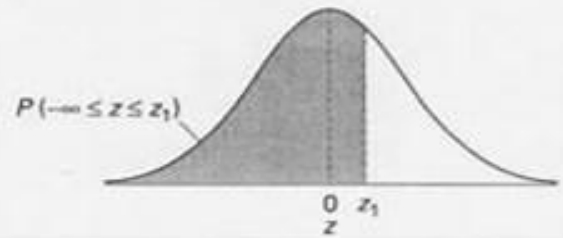
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(b) Table 1. Cumulative distribution function for the standard normal distribution (continued). The table gives the area under the standard normal probability curve between $z = -\infty$ and $z = z_1$.

Example.

If $z_1 = 2.56$, then:

$$P(-\infty < z < 2.56) = 0.99477$$



z_1	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861